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JEE-Main

1st Revision Minor Test

JEE-Mains Type Test paper

Test Date: 15 Dec, 2024 **M.M:300**

TEST INSTRUCTIONS

1. The test is of **3 hours** duration.
2. The test booklet consists of **75 questions**.
3. The maximum marks are **300**.
4. All questions are compulsory.
5. There are three parts in the questions paper consisting of Physics, Chemistry and Mathematics having **25 questions in each part**.

Each Parts Contains –

- 20 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct. All questions are carrying **+4 marks** for right answer and **–1 mark** for wrong answer.
- 05 questions with answer as **numerical value** all questions are carrying **+4 marks** for right answer and **–1 marks** for wrong answers.

Syllabus: Physics-Physics and Measurement, Kinematics, Laws of motion, Friction | Chemistry-Some Basic concepts in chemistry, Atomic structure, Chemical bonding and molecular structure | Math-Sets, Relations and Function, Complex Numbers and Quadratic equations, Sequence and Series

Name of the Candidate (in Capital Letter): ____

Registration No. ____

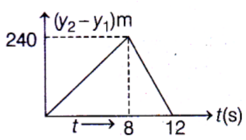
Invigilator Signature

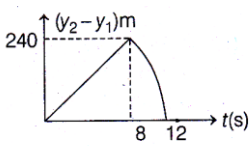
Physics

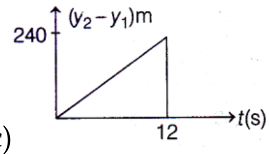
(Single Correct Choice Type)

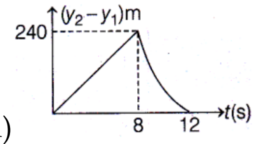
This Section contains **20 multiple choice questions.** Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

1. Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s, respectively. Which of the following graphs best represents the time variation of relative position of the second stone with respect to the first? (Assume stones do not rebound after hitting the ground and neglect air resistance, take $g = 10\,m/s^2$)

(a) 

(b) 

(c) 

(d) 

2. A small block slides without friction down an inclined plane starting from rest. Let s_n be the distance travelled from $t = n-1$ to $t = n$. Then, $\frac{s_n}{s_{n+1}}$ is

(a) $\frac{2n-1}{2n}$ (b) $\frac{2n+1}{2n-1}$ (c) $\frac{2n-1}{2n+1}$ (d) $\frac{2n}{2n+1}$

3. The vernier scale used for measurement has a positive zero error of 0.2 mm. If while taking a measurement it was noted that 0 on the vernier scale lies between 8.5 cm and 8.6 cm, vernier coincidence is 6, then the correct value of measurement is ....cm. (least count = 0.01 cm)

(a) 8.54 cm (b) 8.36 cm (c) 8.56 cm (d) 8.58 cm

4. A person standing on an open ground hears the sound of a jet aeroplane, coming from north at an angle 60^o with ground level. But he finds the aeroplane right vertically above his position. If v is the speed of sound, then speed of the plane is

(a) $\frac{\sqrt{3}}{2}v$ (b) v (c) $\frac{2v}{\sqrt{3}}$ (d) $\frac{v}{2}$

5. A particle is moving Eastwards with a velocity of 5 m/s. In 10 s, the velocity changes to 5 m/s Northwards. The average acceleration in this time is

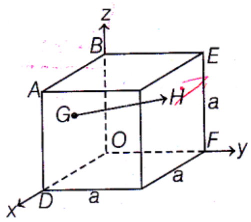
(a) zero (b) $\frac{1}{\sqrt{2}}m/s^2$ towards North-East

(c) $\frac{1}{\sqrt{2}}m/s^2$ towards North-West (d) $\frac{1}{2}m/s^2$ towards North

6. A projectile is given an initial velocity of $(i+2j)m/s$, where, i is along the ground j is along the vertical. If $g = 10\ m/s^2$, then the equation of its trajectory is

(a) $y = x - 5x^2$ (b) $y = 2x - 5x^2$ (c) $4y = 2x - 5x^2$ (d) $4y = 2x - 25x^2$

7. In the cube of side a shown in the figure, the vector from the central point of the face ABOD to the central point of the face BEFO will be

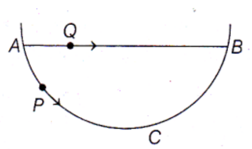


(a) $\frac{1}{2}a(\hat{k}-\hat{i})$ (b) $\frac{1}{2}a(\hat{i}-\hat{k})$ (c) $\frac{1}{2}a(\hat{j}-\hat{i})$ (d) $\frac{1}{2}a(\hat{j}-\hat{k})$

8. A girl standing on road holds her umbrella at 45^o with the vertical to keep the rain away. if she starts running without umbrella with a speed of $15\sqrt{2}\ kmh^{-1}$, the rain drops hit her head vertically. The speed of rain drops with respect to the moving girls is

(a) $30\,kmh^{-1}$ (b) $\frac{25}{\sqrt{2}}kmh^{-1}$ (c) $\frac{30}{\sqrt{2}}kmh^{-1}$ (d) $25\,kmh^{-1}$

9. A particle P is sliding down a frictionless hemispherical bowl. It passes the point A at t = 0. At this instant of time, the horizontal component of its velocity is v. A head Q of the same mass as P is ejected from A at t = 0 along the horizontal string AB, with the speed v. Friction between the bead and the string may be neglected. Let t_P and t_Q be the respective times taken by P and Q to reach the point B.
Then

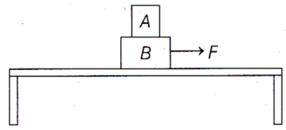


(a) $t_P < t_Q$ (b) $t_P = t_Q$

(c) $t_P > t_Q$ (d) $\frac{t_P}{t_Q} = \frac{\text{length of arc ACB}}{\text{length of chord AB}}$

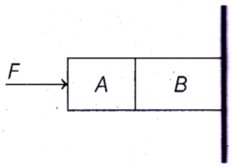
10. A boat which has a speed of 5 km/h in still water crosses a river of width 1 km along the shortest possible path in 15 min. The velocity of the river water in km/h is
(a) 1 (b) 3 (c) 4 (d) $\sqrt{41}$

11. Two blocks A and B of masses $m_A = 1$ kg and $m_B = 3$kg are kept on the table as shown in figure. The coefficient of friction between A and B is 0.2 and between B and the surface of the table is also 0.2. The maximum force F that can be applied on B horizontally, so that the block A does not slide over the block B is [Take, $g = 10 m/s^2$]



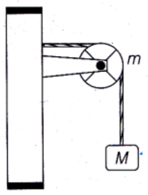
(a) 12 N (b) 16 N (c) 8 N (d) 40 N

12. Given in the figure are two blocks A and B of weight 20 N and 100 N, respectively. These are being pressed against a wall by a force F as shown in figure. If the coefficient of friction between the blocks is 0.1 and between block B and the wall is 0.15, the frictional force applied by the wall in block B is



(a) 100 N (b) 120 N (c) 80 N (d) 150 N

13. A string of negligible mass going over a clamped pulley of mass m supports a block of mass M as shown in the figure. The force on the pulley by the clamp is given by



(a) $\sqrt{2}$ Mg (b) $\sqrt{2}$ mg (c) $g\sqrt{(M+m)^2+m^2}$ (d) $g\sqrt{(M+m)^2+M^2}$

14. A block of mass 0.1 kg is held against a wall applying a horizontal force of 5 N on the block. If the coefficient of friction between the block and the wall is 0.5, the magnitude of the frictional force acting on the block is

(a) 2.5 N (b) 0.98 N (c) 4.9 N (d) 0.49 N

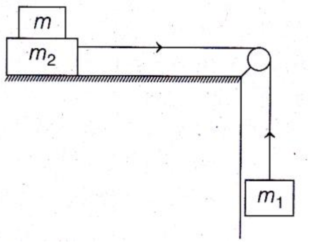
15. A block of mass 2 kg rests on a rough inclined plane making an angle of 30^o with the horizontal. The coefficient of static friction between the block and the plane is 0.7. The frictional force on the block is

(a) 9.8 N (b) $0.7 \times 9.8 \times \sqrt{3}$ N (c) $9.8 \times \sqrt{3}$ N (d) 0.7×9.8 N

16. An insect is at the bottom of a hemispherical ditch of radius 1 m. It crawls up the ditch but starts slipping after it is at height h from the bottom. If the coefficient of friction between the ground and the insect is 0.75, then h is ($g = 10\ ms^{-2}$)

(a) 0.20 m (b) 0.45 m (c) 0.60 m (d) 0.80 m

17. Two masses $m_1 = 5kg$ and $m_2 = 10kg$ connected by an inextensible string over a frictionless pulley, are moving as shown in the figure. The coefficient of friction of horizontal surface is 0.15. The minimum weight m that should be put on top of m_2 to stop the motion is

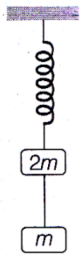


(a) 23.33 kg (b) 18.3 kg (c) 27.3 kg (d) 43.3 kg

18. A disc with a flat small bottom beaker placed on it at a distance R from its center is revolving about an axis passing through the center and perpendicular to its plane with an angular velocity ω. The coefficient of static friction between the bottom of the beaker and the surface of the disc is μ. The breaker will revolve with the disc if

(a) $R \le \frac{\mu g}{2\omega^2}$ (b) $R \le \frac{\mu g}{\omega^2}$ (c) $R \ge \frac{\mu g}{2\omega^2}$ (d) $R \ge \frac{\mu g}{\omega^2}$

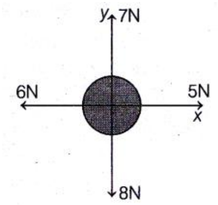
19. System shown in figure is in equilibrium and at rest. The spring and string are massless, now the string is cut. The acceleration of mass 2m and m just after the string is cut will be



(a) g/2 upwards, g = downwards (b) g upwards, g/2 downwards

(c) g upwards, 2g downwards (d) 2g upwards, g downwards

20. For a free body diagram shown in the figure, the four forces are applied in the x and y directions. What additional force must be applied and at what angle with positive X-axis so that the net acceleration of body is zero?



(a) $\sqrt{2}\,N, 45^o$ (b) $\sqrt{2}\,N, 135^o$ (c) $\frac{2}{\sqrt{3}}N, 30^o$ (d) $2N, 45^o$

(Integer Type Questions)

This Section contains **5 Questions.** The answer to each question is a single digit integer ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

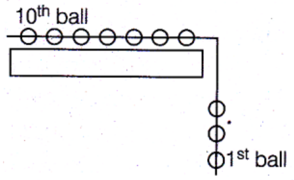
1. A ball is dropped from the top of a 100 m high tower on a planet. In the last $\frac{1}{2}s$ before hitting the ground, it covers a distance of 19 m. Acceleration due to gravity (in ms^{-2}) near the surface on that planet is ....

2. The distance x covered by a particle in one dimensional motion varies with with time t as $x^2 = at^2 + 2bt + c.$ If the acceleration of the particle depends on x as $x^{-n},$ where n is an integer, the value of n is ....

3. A projectile is fired from horizontal ground with speed v and projection angle θ. When the acceleration due to gravity is g, the range of the projectile is d. If at the highest point in its trajectory, the projectile enters a different region where the effective acceleration due to gravity is $g' = \frac{g}{0.81},$ then the new range is $d' = nd.$ The value of n is ....

4. When a body slides down from rest along a smooth inclined plane making an angle of 30^o with time horizontal, it takes time T. When the same body slides down from the rest along a rough inclined plane making the same angle and through the same distance, it takes time $\alpha T,$ where α is a constant greater than 1. The coefficient of friction between the body and the rough plane is $\frac{1}{\sqrt{x}}\left(\frac{\alpha^2 - 1}{\alpha^2}\right).$
where x = ...

5. A system of 10 balls each of mass 2kg are connected via massless and stretchable string. The system is allowed to slip over the edge of a smooth table as shown in figure. Tension on the string between the 7th and 8th ball is .... N when 6th ball just leaves the table.



Chemistry

(Single Correct Choice Type)

This Section contains **20 multiple choice questions.** Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

1. The pollution of SO_2 in air is 10 ppm by volume. The volume of SO_2 per litre of air is:

(a) 10^{-2} mL (b) 10^{-3} mL (c) 10^{-4} mL (d) 10^{-6} mL

2. A compound possess 8% sulphur by mass. The least molecular mass is:

(a) 200 (b) 400 (c) 155 (d) 355

3. 1000 g calcium carbonate solution contains 10 g carbonate. The concentration of solution is:

(a) 10 ppm (b) 100 ppm (c) 1000 ppm (d) 10,000 ppm

4. In the standardization of $Na_2S_2O_3$ using $K_2Cr_2O_7$ by iodometry, the equivalent weight of $K_2Cr_2O_7$ is

(a) Same as mol. wt. (b) $\frac{\text{Mol. wt.}}{2}$

(c) $\frac{\text{Mol. wt.}}{4}$ (d) $\frac{\text{Mol. wt.}}{6}$

5. From the complete decomposition of 20 g $CaCO_3$ at STP the volume of CO_2 obtained is:

(a) 2.24 L (b) 4.48 L (c) 44.8 L (d) 48.4 L

6. When a mixture of Na_2CO_3 and $NaHCO_3$ was heated at 423 K, 112 ml of CO_2 was formed only. What is the % of Na_2CO_3 here in the mixture:

(a) 84% (b) 16% (c) 32% (d) 68%

7. 500 mL of NH_3 contains 6.0×10^{23} molecules at STP. How many molecules are present in 100 mL of CO_3 at STP?

(a) 6×10^{23} (b) 1.5×10^{23} (c) 1.2×10^{23} (d) None of these

8. In the reaction,

$$4NH_3(g)+5O_2(g)\longrightarrow 4NO(g)+6H_2O(l)$$

When 1 mol of ammonia and 1 mole of O_2 are made to react to completion then:

(a) 1.0 mol of H_2O is produced (b) 1.0 mol of NO will be produced

(c) All the ammonia will be consumed (d) All the oxygen will be consumed

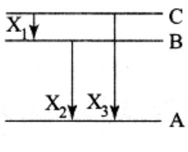
9. Match list-I with list-II and select the correct answer using the codes given below the lists:

	List-I (Metal ions)		**List-II (Magnetic moment)**
1.	Cr^{3+}	(i)	$\sqrt{35}$
2.	Fe^{2+}	(ii)	$\sqrt{30}$
3.	Ni^{2+}	(iii)	$\sqrt{24}$
4.	Mn^{2+}	(iv)	$\sqrt{15}$
		(v)	$\sqrt{8}$

The correct matching is:

	1	**2**	**3**	**4**
(a)	(i)	(iii)	(v)	(iv)
(b)	(ii)	(iii)	(v)	(i)
(c)	(iv)	(iii)	(v)	(i)
(d)	(iv)	(v)	(iii)	(i)

10. Energy levels A, B, C of a certain atom corresponds to increasing values of energy, i.e., $E_A < E_B < E_c$. If X_1, X_2 and X_3 are the wavelengths of radiations corresponding to the transitions C to B, B to A and C to A respectively, which of the following statement is correct?



(a) $X_1+X_2+X_3=0$ (b) $X_3=X_1+X_2$ (c) $X_3^2=X_1^2+X_2^2$ (d) $X_3=\dfrac{X_1X_2}{X_1+X_2}$

11. If the electron of a hydrogen atom is present in the first orbit, the total energy of the electron is

(a) $-e^2/2r$ (b) $-e^2/r$ (c) $-e^2/r^2$ (d) $-e^2/2r^2$

12. The number of nodal planes in a p_x orbital is

(a) 1 (b) 2 (c) 3 (d) 0

13. The quantum number 'm' of a free gaseous atom is associated with:

(a) The effective volume of the orbital

(b) The shape of the orbital

(c) The spatial orientation of the orbital

(d) The energy of the orbital in the absence of a magnetic field

14. Which of the following species have undistorted octahedral structures?

(1) SF_6 (2) PF_6^- (3) SiF_6^{2-} (4) XeF_6

Select the correct answer using the codes given below:

(a) 1, 3 and 4 (b) 1, 2 and 3 (c) 1, 2 and 4 (d) 2, 3 and 4

15. Consider the given statements about the molecule $(H_3C)_2CH-CH=CH-C\equiv C-CH=CH_2$.

1. Three carbon atoms are sp^3 hybridized
2. Three carbon atoms are sp^2 hybridized
3. Two carbon atoms are sp hybridized Of three statements

(a) 1 and 2 are correct (b) 1 and 3 are correct

(c) 2 and 3 are correct (d) 1, 2 and 3 are correct

16. Match the following:

	List-I (Species)		**List-II (Bond Order)**
1.	O_2^{2+}	(1)	1.0
2.	O_2	(2)	2.0
3.	F_2	(3)	2.5
4.	O_2^+	(4)	3.0

The correct matching is:

	1	**2**	**3**	**4**
(a)	4	1	2	2
(b)	2	3	1	4
(c)	4	2	1	3
(d)	3	4	1	2

17. According to molecular orbital theory which of the following statement about the magnetic character and bond order is correct regarding O_2^+?

(a) paramagnetic and bond order < O_2 (b) paramagnetic and bond order > O_2

(c) diamagnetic and bond order < O_2 (d) diamagnetic and bond order > O_2

18. Pick out the isoelectronic structure from the following:

1. CH_3 2. H_3O^+ 3. NH_3 4. CH_2

(a) 1 and 2 (b) 3 and 4 (c) 1 and 3 (d) 2, 3 and 4

19. If the molecules of HCl was totally polar, the expected value of dipole moment was 6.12 D but the experimental value of dipole moment calculated was 1.03 D. Calculate the percentage ionic character.

(a) 0 (b) 17 (c) 50 (d) 90

20. Specify the coordination geometry and hybridiza-tion of N and B atoms in a 1 : 1 complex of BF_3 and NH_3.

(a) N : tetrahedral, sp; B : tetrahedral, sp^3 (b) N : pyramidal, sp^3; B : pyramidal, sp^3

(c) N : pyramidal, sp^3; B : planar, sp^2 (d) N : pyramidal, sp^3; B : tetrahedral, sp^3

(Integer Type Questions)

This Section contains 5 **Questions.** The answer to each question is a single digit integer ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

1. The equivalent weight of phosphoric acid (H_3PO_4) in the reaction:

$$NaOH + H_3PO_4 \longrightarrow NaH_2PO_4 + H_2O \text{ is}$$

2. A monoenergetic electron beam with a de Broglie wavelength of x Å is accelerated till its wavelength is halved. By what factor is its kinetic energy changed?

3. In hydrogen atom, an orbit has a diameter of about 16.92 A. What is the maximum number of electrons that can be accommodated?

4. Ratio of radii of second and first Bohr orbits of H atom is

5. The atomic number of an element is 35. What is the total number of electrons present in all the p orbitals of the ground state atom of that element?

Mathematics

(Single Correct Choice Type)

This Section contains **20 multiple choice questions.** Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

1. Let the range of the function $f(x)=\dfrac{1}{2+\sin 3x+\cos 3x}, x\in IR$ be [a,b]. If α and β are respectively the A.M. and G.M. of a and b , then $\dfrac{\alpha}{\beta}$ is equal to:

(a) $\sqrt{2}$ (b) 2 (c) $\sqrt{\pi}$ (d) π

2. If the domain of the funciton $f(x)=\dfrac{\sqrt{x^2-25}}{(4-x^2)}+\log_{10}(x^2+2x-15)$ is $(-\infty,\alpha)\cup[\beta,\infty]$, then $\alpha^2+\beta^3$ is equal to

(a) 140 (b) 175 (c) 125 (d) 150

3. Let $f:R-\{0,1\}\to R$ be a functoin such that $f(x)+f\left(\dfrac{1}{1-x}\right)=1+x$. Then $f(2)$ is equal to

(a) $\dfrac{9}{2}$ (b) $\dfrac{7}{4}$ (c) $\dfrac{9}{4}$ (d) $\dfrac{7}{3}$

4. Let $f:R\to R$ and $g:R\to R$ be defined as $f(x)=\begin{cases}\log_e x, & x>0\\ e^{-x}, & x\le 0\end{cases}$ and $g(x)=\begin{cases}x, & x\ge 0\\ e^{x}, & x<0\end{cases}$. Then gof $g:R\to R$ is :

(a) one – one but not onto (b) neither one – one nor onto
(c) onto but not one – one (d) both one – one and onto

5. Let $f(x)=\begin{cases}x-1, x\, is\, even\\ 2x, x\, is\, odd\end{cases}$ $x\in N$. If for some $a\in N$, $f(f(f(a)))=21$, then $\lim\limits_{x\to a^-}\left\{\dfrac{|x|^3}{a}-\left[\dfrac{x}{a}\right]\right\}$, where [t] denotes the greatest integer less than or equalto t, is equalto:

(a) 169 (b) 121 (c) 225 (d) 144

6. Let r and θ respectively be the modulus and amplitude of the complex numbers $z=2-i\left(2\tan\dfrac{5\pi}{8}\right)$, then (r,θ) is equal to

(a) $\left(2\sec\dfrac{11\pi}{8},\dfrac{11\pi}{8}\right)$ (b) $\left(2\sec\dfrac{5\pi}{8},\dfrac{3\pi}{8}\right)$ (c) $\left(2\sec\dfrac{3\pi}{8},\dfrac{5\pi}{8}\right)$ (d) $\left(2\sec\dfrac{3\pi}{8},\dfrac{3\pi}{8}\right)$

7. If $z=\dfrac{1}{2}-2i$ is such that $|z+1|=\alpha z+\beta(1+i), i=\sqrt{-1}$ and $\alpha,\beta\in R$, then $\alpha+\beta$ is equal to

(a) -1 (b) -4 (c) 2 (d) 3

8. If 2 and 6 are the roots of the equation $ax^2+bx+1=0$, then the quadratic equation, whose roots are $\frac{1}{2a+b}$ and $\frac{1}{6a+b}$, is :

(a) $2x^2+11x+12=0$ (b) $4x^2+14x+12=0$ (c) $x^2+10x+16=0$ (d) $x^2+8x+12=0$

9. Let $S=\{x:x\in R\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4}+\left(\sqrt{3}-\sqrt{2}\right)^{x^2-4}=10\}$. Then n(S) is euqal to

(a) 4 (b) 0 (c) 6 (d) 2

10. If α and β are roots of the equation $x^2+px+\frac{3p}{4}=0$, such that $|\alpha-\beta|=\sqrt{10}$, then p belongs to the set:

(a) {2, -5} (b) {-3,2} (c) {-2,5} (d) {3,-5}

11. Let α,β be two roots of the equation $x^2+(20)^{1/4}x+(5)^{1/2}=0$. Then $\alpha^8+\beta^8$ is equal to

(a) 10 (b) 100 (c) 50 (d) 160

12. Let α and β be the sum and the product of all the non-zero soltuoins of the equation $\left(\bar{z}\right)^2+|z|=0, z\in C$. Then $4(\alpha^2+\beta^2)$ is equal to :

(a) 6 (b) 4 (c) 8 (d) 2

13. If a,b, and u,v,w are copmplex numbers representing the vertices of two triangles such that $c=(1-r)a+rb$ and $w=(1-r)u+rv$, where r is a complex number , then the two triangles

(a) have the same area (b) are similar

(c) are congruent (d) none of these

14. The inequality $|z-4|<|z-2|$ represents the region given by

(a) $\text{Re}(z)\geq 0$ (b) $\text{Re}(z)<0$ (c) $\text{Re}(z)>0$ (d) None of these

15. The 20th term from the end of the progression $20,19\frac{1}{4},18\frac{1}{2},17\frac{3}{4},\ldots,-129\frac{1}{4}$ is : -

(a) -118 (b) -110 (c) -115 (d) -100

16. Let S_n denote the sum of the first n terms of an arithmetic progresson. If $S_{10}=390$ and the ratio of the tenth and the fifth terms is 15 :7 , then $S_{15}-S_5$ is equalto

(a) 800 (b) 890 (c) 790 (d) 690

17. If each term of a geometric progression $a_1,a_2,a_3,\ldots$ with $a_1=\frac{1}{8}$ and $a_2\neq a_1$, is the arithmetic mean of the next two terms and $S_n=a_1+a_2+\ldots+a_n$, then $S_{20}-S_{18}$ is equal to

(a) 2^{18} (b) 2^{15} (c) -2^{18} (d) -2^{15}

18. Let $f(x)$ be a function such that $f(x+y)=f(x).f(y)$ for all $x,y,\in N$. If $f(1)=3$ and $\sum_{k=1}^{n}f(k)=3279$, then the value of n is

(a) 6 (b) 8 (c) 7 (d) 9

19. Let α and β be the roots of the quadratic equaiton $x^2 \sin\theta - x(\sin\theta\cos\theta + 1) + \cos\theta = 0 (0 < \theta < 45^o)$, and $\alpha < \beta$. Then $\sum_{n=0}^{\infty}\left(\alpha^n + \frac{(-1)^n}{\beta^n}\right)$ is equal to :

(a) $\frac{1}{1-\cos\theta} - \frac{1}{1+\sin\theta}$
(b) $\frac{1}{1+\cos\theta} + \frac{1}{1-\sin\theta}$
(c) $\frac{1}{1-\cos\theta} + \frac{1}{1+\sin\theta}$
(d) $\frac{1}{1+\cos\theta} - \frac{1}{1-\sin\theta}$

20. If $2\tan^2\theta - 5\sec\theta = 1$ has exactly 7 solutions in the interval $\left[0, \frac{n\pi}{2}\right]$, for the least value of $n \in N$ then $\sum_{k=1}^{n} \frac{k}{2^k}$ is equal to :

(a) $\frac{1}{2^{15}}\left(2^{14} - 14\right)$
(b) $\frac{1}{2^{14}}\left(2^{15} - 15\right)$
(c) $1 - \frac{15}{2^{13}}$
(d) $\frac{1}{2^{13}}\left(2^{14} - 15\right)$

(Integer Type Questions)

This Section contains **5 Questions.** The answer to each question is a single digit integer ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

1. If $S = \{a \in R : |2a - 1| = 3[a] + 2\{a\}\}$, where [t] denotes the greatest integer less than or equal to t and {t} represents the fraction part of t, then $72\sum_{a \in S} a$ is equal to .

2. For $x \in R$, then number of real roots of the equation $3x^2 - 4|x^2 - 1| + x - 1 = 0$ is .

3. The least positive value of 'a' for which the equation $2x^2 + (a-10)x + \frac{33}{2} = 2a$ has real root is ____.

4. The smallest value of k, for which both the roots of the equation $x^2 - 8kx + 16(k^2 - k + 1) = 0$ are real, distinct and have values at least 4, is

5. Suppose that all the terms of an arithmetic progression (A.P.) are natural numbers. Ifthe ratio of the sum of the first seven terms to the sum of the first eleven terms is 6:11 and the seventh term lies in between 130 and 140, then the common difference of this A.P. is

Date- 15-12-2024 **Answer – key**

1st Revision Minor JEE-Main Test (Main Type)

Physics

1. d
2. c
3. a
4. d
5. c
6. b
7. c
8. c
9. a
10. b
11. b
12. b
13. d
14. b
15. a
16. a
17. a
18. b
19. a
20. a

Integer

1. 8
2. 3
3. 0.95
4. 3
5. 36

Chemistry

1. a
2. b
3. d
4. d
5. b
6. b
7. c
8. d
9. c
10. d
11. a
12. a
13. c
14. b
15. b
16. c
17. b
18. d
19. b
20. a

Integer

1. 98
2. 4
3. 32
4. 4
5. 17

Maths

1. a
2. d
3. c
4. b
5. d
6. d
7. d
8. d
9. a
10. c
11. c
12. b
13. b
14. d
15. c
16. c
17. d
18. c
19. c
20. d

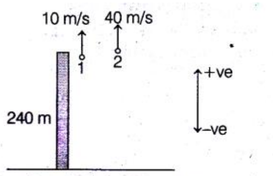
Integer

1. 18
2. 4
3. 8
4. 2
5. 9

1st Revision Minor JEE-Main Test (Main Type)

PHYSICS

1. (d)



Let us first find the time of collision of two particles with the ground

$$S = ut + \frac{1}{2}at^2$$

$$-240 = 10t_1 - \frac{1}{2} \times 10 \times t_1^2$$

Solving, we get positive value of $t_1 = 8\,s$

Similarly, $-240 = 40t_2 - \frac{1}{2} \times 10 \times t_2^2$

On solving this equation, we get positive value of $t_2 = 12\,s$

From 0 to 8 s, both particles are moving under gravity. Their absolute accelerations are same (equal to g). So, relative acceleration is zero or relative motion is uniform. So, relative position will change (or increase) linearly. At 8 sec, first particle strikcs with ground. Its acceleration has become zero. The second particle has reached its initial position and is moving downwards so, relative position will now decrease. The particle is still in air. So its acceleration is g, or relative acceleration is g.

Hence, graph is parabola.

2. (c)

Distance travelled in n^{th} second is,

$$s_n = u + an - \frac{1}{2}a$$

Given, $u = 0$

$$\therefore \quad \frac{s_n}{s_{n+1}} = \frac{an - \frac{1}{2}a}{a(n+1) - \frac{1}{2}a} = \frac{2n-1}{2n+1}$$

∴ Correct option is (c).

3. (a)

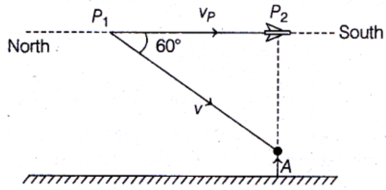
Correct measured value

$= \text{MSR} + (\text{VSD} \times \text{LC}) - \text{Zero error}$

$= 8.5 + (6 \times 0.01) - 0.2 \times 10^{-1} = 8.54\text{ cm}$

4. (d)

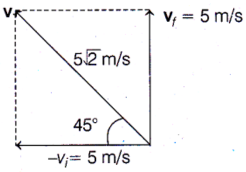
Let P_1 be the position of plane at $t = 0$, when sound waves started towards person A and P_2 is the position of plane observed at time instant t as shown in the figure below.



$$\cos 60° = \frac{P_1P_2}{P_1A} = \frac{v_P \times t}{v \times t}$$

$$\frac{1}{2} = \frac{v_P}{v} \Rightarrow v_P = \frac{v}{2}$$

5. (c)



$$\mathbf{a}_{av} = \frac{\Delta \mathbf{v}}{\Delta t} = \frac{\mathbf{v}_f - \mathbf{v}_i}{\Delta t} = \frac{\mathbf{v}_f + (-\mathbf{v}_i)}{\Delta t}$$

$\Delta \mathbf{v} = 5\sqrt{2}$ m/s in North-West direction.

$$\mathbf{a}_{av} = \frac{5\sqrt{2}}{10} = \frac{1}{\sqrt{2}}\text{ m/s}^2$$

(in North-West direction)

6. (b)

Initial velocity $= (\mathbf{i} + 2\mathbf{j})$ m/s

Magnitude of initial velocity,

$$u = \sqrt{u_x^2 + u_y^2} = \sqrt{(1)^2 + (2)^2}$$

$$= \sqrt{5}\text{ m/s}$$

Equation of trajectory of projectile is

$$y = x\tan\theta - \frac{gx^2}{2u^2}(1 + \tan^2\theta)$$

$$\left[\tan\theta = \frac{u_y}{u_x} = \frac{2}{1} = 2\right]$$

Substituting the values, we get

$$\therefore \quad y = x \times 2 - \frac{10(x)^2}{2(\sqrt{5})^2}[1 + (2)^2]$$

or $\quad y = 2x - 5x^2$

7. (c)

From figure, the position vector of **G**,

$$\mathbf{r}_G = \frac{a}{2}\hat{\mathbf{i}} + \frac{a}{2}\hat{\mathbf{k}}$$

the position vector of H,

$$\mathbf{r}_H = \frac{a}{2}\hat{\mathbf{j}} + \frac{a}{2}\hat{\mathbf{k}}$$

$$\therefore \mathbf{r}_H - \mathbf{r}_G = \left(\frac{a}{2}\hat{\mathbf{j}} + \frac{a}{2}\hat{\mathbf{k}}\right) - \left(\frac{a}{2}\hat{\mathbf{i}} + \frac{a}{2}\hat{\mathbf{k}}\right)$$

$$= \frac{a}{2}(\hat{\mathbf{j}} - \hat{\mathbf{i}})$$

8. (c)

In ΔABC

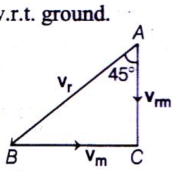
Clearly, $\tan 45^\circ = \frac{|v_{rm}|}{|v_m|}$

where, v_{rm} is velocity of rain w.r.t. girl,
v_m is velocity of girl w.r.t. ground,
v_r is velocity of rain w.r.t. ground.

$\Rightarrow \quad v_{rm} = v_m$

$\Rightarrow \quad v_{rm} = 15\sqrt{2} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{30}{\sqrt{2}}$ km/h.

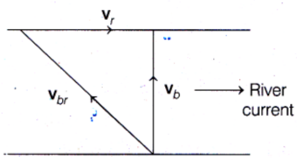


9. (a)

For particle P, motion between AC will be an accelerated one while between CB a retarded one. But in any case horizontal component of its velocity will be greater than or equal to v. On the other hand, in case of particle Q, it is always equal to v. Horizontal displacement for both the particles is same. Therefore, $t_P < t_Q$.

10. (b)

Shortest possible path comes when absolute velocity of boatman with respect to ground is perpendicular to river current as shown in figure.



$$t = \frac{w}{v_b} = \frac{w}{\sqrt{v_{br}^2 - v_r^2}}$$

w-width (1 km)

$$t = 15 \text{ min} = \frac{1}{4}\text{ h} \Rightarrow \frac{1}{4} = \frac{1}{\sqrt{25 - v_r^2}}$$

Solving this equation, we get
$v_r = 3$ km/h

11. (b)

Acceleration a of system of blocks A and B is

$$a = \frac{\text{Net force}}{\text{Total mass}}$$

$$= \frac{F - f_1}{m_A + m_B}$$

where, f_1 = friction between B and the surface

$$= \mu(m_A + m_B)g$$

So, $$a = \frac{F - \mu(m_A + m_B)g}{(m_A + m_B)} \quad \ldots\text{(i)}$$

Here, $\mu = 0.2$,
$m_A = 1\,\text{kg}$, $m_B = 3\,\text{kg}$, $g = 10\,\text{ms}^{-2}$

Substituting the above values in Eq. (i), we have

$$a = \frac{F - 0.2(1+3)\times 10}{1+3}$$

$$a = \frac{F-8}{4} \quad \ldots\text{(ii)}$$

Block A moves due to friction between A and B (say f_2).
We consider the limiting case,

$m_A a = f_2$

$\Rightarrow \quad m_A a = \mu(m_A)g$

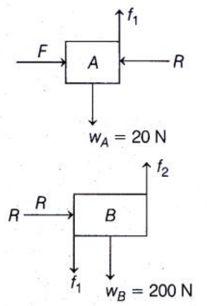
$\Rightarrow \quad a = \mu g = 0.2 \times 10 = 2\,\text{ms}^{-2} \ldots\text{(iii)}$

Putting the value of a from Eq. (iii) into Eq. (ii), we get

$$\frac{F-8}{4} = 2 \Rightarrow F = 16\,\text{N}$$

12. (b)

NOTE It is not given in the question, but assuming that both blocks are in equilibrium. The free body diagram of two blocks is as shown below,



Reaction force, R = applied force F
For vertical equilibrium of A;
f_1 = friction between two blocks
$= W_A = 20\,\text{N}$
For vertical equilibrium of B;
f_2 = friction between block B and wall
$= W_B + f_1 = 100 + 20 = 120\,\text{N}$

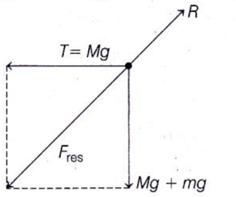
13. (d)

Free body diagram of the pulley (in equilibrium)

Resultant F_{res} of these three forces is

$F_{res} = g\sqrt{(M+m)^2 + M^2}$

Therefore, the reaction force R is equal and opposite to F_{res} as shown in figure.

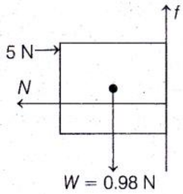


$\therefore \quad R = (\sqrt{(M+m)^2 + M^2})\,g$

14. (b)

$N = 5\text{ N}$

$(f)_{max} = \mu N = (0.5)(5) = 2.5\text{ N}$



For vertical equilibrium of the block,

$f = mg = 0.98\text{ N} < (f)_{max}$

15. (a)

Since, the block is at rest

Thus, $f \geq mg\sin\theta$

$\therefore$ Force of friction is $f = mg\sin\theta$

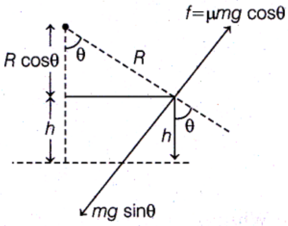
$2 \times 9.8 \times \frac{1}{2} = 9.8\text{ N}$

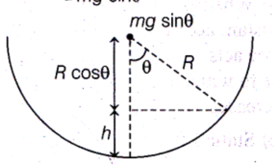
16. (a)

For balancing,

$mg\sin\theta = f = \mu mg\cos\theta$

$\Rightarrow \tan\theta = \mu = \frac{3}{4} = 0.75$



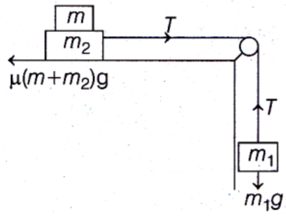


$$h = R - R\cos\theta = R - R\left(\frac{4}{5}\right) = \frac{R}{5}$$

$\therefore \quad h = \frac{R}{5} = 0.2\text{ m} \quad [\because \text{radius, } R = 1\text{ m}]$

17. (a)

None of the four options are correct.



Substituting, $m_1 = 5\text{ kg}$ and $m_2 = 10\text{ kg}$

We get, $\mu(10+m)g \geq 5g$

$10 + m \geq \frac{5}{0.15}$

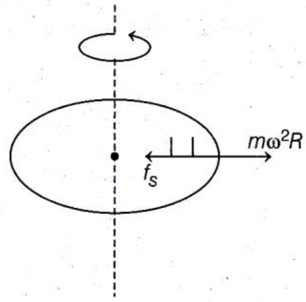
$\therefore \quad m \geq 23.33\text{ kg}$

18. (b)

Static friction $f_s = m\omega^2 R$

So, R will be maximum, when $f_s = f_{lim}$

Therefore,



$f_{lim} = m\omega^2 R_{max} \Rightarrow \mu mg = m\omega^2 R_{max}$

$R_{max} = \frac{\mu g}{\omega^2}$

So, $\quad R \leq \frac{\mu g}{\omega^2}$

19. (a)

Initially under equilibrium of mass m

$T = mg$

Now, the string is cut. Therefore,

$T = mg$, force is decreased on mass m upwards and downwards on mass $2m$.

$\therefore \quad a_m = \frac{mg}{m} = g \quad$ (downwards)

and $\quad a_{2m} = \frac{mg}{2m} = \frac{g}{2} \quad$ (upwards)

20. (a)

For net acceleration; $a_{net} = 0$

Here $a_{net} = \frac{F_{net}}{M} = 0, F_{net} = 0$

Let addition force required be = **F**

$\mathbf{F} + 5\hat{\mathbf{i}} - 6\hat{\mathbf{i}} + 7\hat{\mathbf{j}} - 8\hat{\mathbf{j}} = 0$

$\Rightarrow \mathbf{F} = \hat{\mathbf{i}} + \hat{\mathbf{j}}, |\mathbf{F}| = \sqrt{1^2 + 1^2} = \sqrt{2}\text{ N}$

Angle with X-axis :

$\tan\theta = \frac{y\text{ component}}{x\text{ component}} = \frac{1}{1}$

So, $\theta = \tan^{-1}(1) = 45°$

Integer Type

1. (8)

Given, Tower height = 100 m

Let the ball take time t to reach the ground

Using, $S = ut + \frac{1}{2}gt^2$

$\Rightarrow S = 0 \times t + \frac{1}{2}gt^2$ $[u = 0]$

$\Rightarrow 200 = gt^2$ $[\because 2S = 100\text{ m}]$

$\Rightarrow t = \sqrt{\frac{200}{g}}$ ...(i)

In last $\frac{1}{2}$ s, body travels a distance of 19 m, so in $\left(t - \frac{1}{2}\right)$ distance travelled = 81 m

Now, $\frac{1}{2}g\left(t - \frac{1}{2}\right)^2 = 81$

$\Rightarrow g\left(t - \frac{1}{2}\right)^2 = 81 \times 2$

$\Rightarrow \left(t - \frac{1}{2}\right) = \sqrt{\frac{81 \times 2}{g}}$

$\therefore \frac{1}{2} = \frac{1}{\sqrt{g}}(\sqrt{200} - \sqrt{81 \times 2})$

Using Eq. (i)

$\Rightarrow \sqrt{g} = 2(10\sqrt{2} - 9\sqrt{2})$

$\Rightarrow \sqrt{g} = 2\sqrt{2}$

$\therefore g = 8\text{ m/s}^2$

2. (3)

Distance X varies with time t as

$x^2 = at^2 + 2bt + c$

$\Rightarrow 2x\frac{dx}{dt} = 2at + 2b \Rightarrow x\frac{dx}{dt} = at + b$

$\Rightarrow \frac{dx}{dt} = \frac{(at + b)}{x}$

$\Rightarrow x\frac{d^2x}{dt^2} + \left(\frac{dx}{dt}\right)^2 = a$

$\Rightarrow \frac{d^2x}{dt^2} = \frac{a - \left(\frac{dx}{dt}\right)^2}{x} = \frac{a - \left(\frac{at + b}{x}\right)^2}{x}$

$\Rightarrow \frac{ax^2 - (at + b)^2}{x^2} = \frac{ac - b^2}{x^3}$

$\Rightarrow a \propto x^{-3}$

Hence, $n = 3$

3. (0.95)

After entering in new region, time taken by projectile to reach ground is given as

$t = \sqrt{\frac{2h_{max}}{g_{eff}}} = \sqrt{\frac{2 \times 0.81 \times u^2\sin^2\theta}{g \times 2g}}$

$= 0.9\frac{u\sin\theta}{g}$ $\left[\because g' = \frac{g}{0.81}\right]$

So, horizontal displacement done by projectile in the new region is given by

$x = 0.9 \times \frac{u\sin\theta}{g} \times u\cos\theta = 0.9\left(\frac{d}{2}\right)$

[$\because d$ is the range of projectile in gravity g]

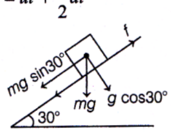
Now, new range $= \frac{d}{2} + \frac{0.9d}{2} = 0.95d$

[As on heightest point of projectile, range will be half]

4. (3)

Acceleration on smooth inclined plane $a = g\sin 30° = g/2$

Using $S = ut + \frac{1}{2}at^2$



$\Rightarrow S = \frac{1}{2}\frac{g}{2}T^2 = \frac{g}{4}T^2$ ...(i) $(\because u = 0)$

Accelerations on rough inclined plane

$a = g\sin 30° - \mu g\cos 30° = \frac{g}{2} - \frac{\mu g\sqrt{3}}{2}$

($\because \mu g\cos 30°$ due to friction)

$\Rightarrow a = \frac{g}{2}(1 - \mu\sqrt{3})$

Using again $S = ut + \frac{1}{2}at^2$

$\Rightarrow S = \frac{1}{4}g(1 - \sqrt{3}\mu)(\alpha T)^2$ ... (ii)

By (i) and (ii)

$\Rightarrow \frac{1}{4}gT^2 = \frac{1}{4}g(1 - \sqrt{3}\mu)\alpha^2T^2$

$\Rightarrow 1 - \sqrt{3}\mu = \frac{1}{\alpha^2}$

$\Rightarrow \mu = \left(\frac{\alpha^2 - 1}{\alpha^2}\right)\frac{1}{\sqrt{3}} \Rightarrow x = 3.00$

5. (36)

We have, acceleration of system as

$a = \frac{F_{net}}{\text{total mass}}$

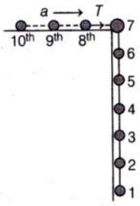
$a = \frac{6mg}{10m} = \frac{3g}{5}$

taking 8, 9, 10 together

$T = 3ma = 3m \times \frac{3g}{5}$

$= \frac{3 \times 2 \times 3 \times 10}{5}$

$= 36\text{N}$



Mathematics

1. Sol.(a)

(a) $f(x) = \frac{1}{2 + \sin 3x + \cos 3x} \in \left[\frac{1}{2+\sqrt{2}}, \frac{1}{2-\sqrt{2}}\right], \forall x \in R$

$\frac{\alpha}{\beta} = \frac{a+b}{2\sqrt{ab}} = \frac{1}{2}\left(\sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}}\right)$

$= \frac{1}{2}\left(\sqrt{\frac{2-\sqrt{2}}{2+\sqrt{2}}} + \sqrt{\frac{2+\sqrt{2}}{2-\sqrt{2}}}\right)$

$= \frac{(2-\sqrt{2}) + (2+\sqrt{2})}{2 \times \sqrt{2}} = \sqrt{2}$

2. Sol.(d)

(d) $f(x) = \frac{\sqrt{x^2 - 25}}{4 - x^2} + \log_{10}(x^2 + 2x - 15)$

for domain: $x^2 - 25 \geq 0 \Rightarrow x \in (-\infty, -5) \cup (5, \infty)$

$4 - x^2 \neq 0 \Rightarrow x \neq \{-2, 2\}$

and $x^2 + 2x - 15 > 0 \Rightarrow (x+5)(x-3) > 0$

$\Rightarrow x \in (-\infty, -5) \cup (3, \infty)$

$\therefore x \in (-\infty, -5) \cup [5, \infty)$ is the domain

Hence $\alpha = -5; \beta = 5 \Rightarrow \alpha^2 + \beta^3 = 150$

3. Sol.(c)

(c) Given equation is $f(x) + f\left(\frac{1}{1-x}\right) = 1 + x$

Put, $x = 2 \Rightarrow f(2) + f(-1) = 3$ ... (i)

Put, $x = -1 \Rightarrow f(-1) + f\left(\frac{1}{2}\right) = 0$ ... (ii)

Now, put, $x = \frac{1}{2} \Rightarrow f\left(\frac{1}{2}\right) + f(2) = \frac{3}{2}$ ... (iii)

After solving (i), (ii) and (iii) we get

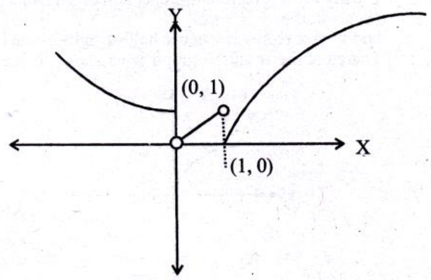
$f(2) = \frac{9}{4}$

4. Sol.(b)

$g(f(x))$ can be written as,

$g(f(x)) = \begin{cases} f(x), & f(x) \geq 0 \\ e^{f(x)}, & f(x) < 0 \end{cases}$

$\Rightarrow \quad g(f(x)) = \begin{cases} e^{-x}, & -\infty < x \leq 0 \\ e^{\ln x}, & 0 < x < 1 \\ \ln x, & 1 \leq x < \infty \end{cases}$



From graph of $g(f(x))$. it is clear that $g(f(x))$ is many one into function.

5. Sol.(b)

$f(x) = \begin{cases} x - 1; & x = \text{even} \\ 2x; & x = \text{odd} \end{cases}, f(f(f(a))) = 21$

Case 1. If a is even.

$f(a) = a - 1$ (odd), $f(f(a)) = 2(a-1)$ (even)

$f(f(f(a))) = 2a - 3 = 21 \Rightarrow a = 12$

Case 2. If a is odd

$f(a) = 2a$(even)

$f(f(a)) = 2a - 1 =$ (odd)

$f(f(f(a))) = 4a - 2 = 21 \Rightarrow a = \frac{23}{4}$

Which is not possible because $a \in N$.

Hence $a = 12$

Now, $\lim_{x \to 12}\left(\frac{|x|^3}{2} - \left[\frac{x}{12}\right]\right)$

$= \lim_{x \to 12^-} \frac{|x|^3}{12} - \lim_{x \to 12^-}\left[\frac{x}{12}\right] = 144 - 0 = 144$

6. Sol.(d)

(d) $z = 2 - i\left(2\tan\frac{5\pi}{8}\right) = x + iy \text{ (let)}$

$x = 2,\ y = -2\tan\frac{5\pi}{8}$

$r = \sqrt{x^2 + y^2}\ \ \&\ \theta = \tan^{-1}\frac{y}{x}$

$r = \sqrt{(2)^2 + \left(2\tan\frac{5\pi}{8}\right)^2}$

$= \left|2\sec\frac{5\pi}{8}\right| = \left|2\sec\left(\pi - \frac{3\pi}{8}\right)\right|$

$= 2\sec\frac{3\pi}{8}$

$\&\ \theta = \tan^{-1}\left(\frac{-2\tan\frac{5\pi}{8}}{2}\right) = \tan^{-1}\left(\tan\left(\pi - \frac{5\pi}{8}\right)\right)$

$= \frac{3\pi}{8}$

7. Sol.(d)

(d) $z = \frac{1}{2} - 2i$ ...(i)

$|z - 1| = \alpha z + \beta(1 + i)$ ...(ii)

Substitute (i) in (ii)

$\left|\frac{3}{2} - 2i\right| = \frac{\alpha}{2} - 2\alpha i + \beta + \beta i$

$\left|\frac{3}{2} - 2i\right| = \left(\frac{\alpha}{2} + \beta\right) + (\beta - 2\alpha)i$

$\beta = 2\alpha$ and $\frac{\alpha}{2} + \beta = \sqrt{\frac{9}{4} + 4} = \frac{5}{2}$

On solving, we get $\alpha = 1, \beta = 2$

$\alpha - \beta = 3$

8. Sol.(d)

$\text{Sum} = 8 = -\frac{b}{a}$

$\text{Product} = 12 = \frac{1}{a}$

$\Rightarrow a = \frac{1}{12},\ b = -\frac{2}{3}$

$2a + b = \frac{2}{12} - \frac{2}{3} = -\frac{1}{2} \Rightarrow \frac{1}{2a + b} = -2$

$6a + b = \frac{6}{12} - \frac{2}{3} = -\frac{1}{6} \Rightarrow \frac{1}{6a + b} = -6$

sum $= -8$

Product $= 12$

$x^2 + 8x + 12 = 0.$

9. Sol.(a)

Let $(\sqrt{3} + \sqrt{2})^{x^2 - 4} = t,\ (\sqrt{3} - \sqrt{2})^{x^2 - 4} = \frac{1}{t}$

$t + \frac{1}{t} = 10 \Rightarrow t^2 - 10t + 1 = 0 \Rightarrow t = 5 \pm 2\sqrt{6}$

$\Rightarrow (\sqrt{3} + \sqrt{2})^{x^2 - 4} = 5 + 2\sqrt{6}, 5 - 2\sqrt{6}$

$\Rightarrow x^2 - 4 = 2, -2$ or $x^2 = 6, 2 \Rightarrow x = \pm\sqrt{2}, \pm\sqrt{6}$

There are total 4 solutions

10. Sol.(c)

Given quadratic eqn. is

$x^2 + px + \frac{3p}{4} = 0$

So, $\alpha + \beta = -p,\ \alpha\beta = \frac{3p}{4}$

Now, given $|\alpha - \beta| = \sqrt{10} \Rightarrow \alpha - \beta = \pm\sqrt{10}$

$\Rightarrow (\alpha - \beta)^2 = 10 \Rightarrow \alpha^2 + \beta^2 - 2\alpha\beta = 10$

$\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = 10$

$\Rightarrow p^2 - 4 \times \frac{3p}{4} = 10 \Rightarrow p^2 - 3p - 10 = 0$

$\Rightarrow p = -2, 5 \Rightarrow p \in \{-2, 5\}$

11. Sol.(c)

$x^2 + 5^{\frac{1}{2}} = -(20)^{\frac{1}{4}}x \Rightarrow \left(x^2 + \sqrt{5}\right)^2 = \sqrt{20}x^2$

$\Rightarrow x^4 + 5 + 2\sqrt{5}x^2 = 2\sqrt{5}x^2 \Rightarrow x^4 = -5 \Rightarrow x^8 = 25$

So, $\alpha^8 + \beta^8 = 50$

12. Sol.(b)

$z = x + iy$

$\bar{z} = x - iy$

$\bar{z}^2 = x^2 - y^2 - 2ixy$

$\Rightarrow x^2 - y^2 - 2ixy + \sqrt{x^2 + y^2} = 0$

$\Rightarrow x = 0$ or $y = 0$

$-y^2 + |y| = 0$ $\quad x^2 + |x| = 0$

$|y| = |y|^2$ $\quad \Rightarrow x = 0$

$y = 0, \pm 1$

Hence, $z = \pm i$ are the non-zero solution of the given equation

$\Rightarrow \alpha = i - i = 0$

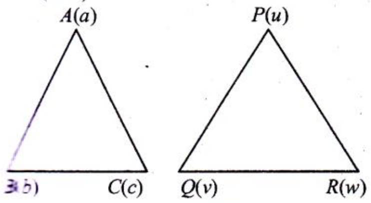
$\beta = i(-i) = 1$

$4(\alpha^2 + \beta^2) = 4(0 + 1) = 4.$

13. Sol.(b)

Let ABC be the Δ whose vertices are represented by complex numbers a, b, c and PQR be the Δ with whose vertices are represented by complex numbers u, v, w.

Then $c = (1-r)a + rb$



$\Rightarrow c - a = r(b-a) \Rightarrow \dfrac{c-a}{b-a} = r$ ....(i)

$\Rightarrow w = (1-r)u + rv \Rightarrow \dfrac{w-u}{v-u} = r$ ....(ii)

From (i) and (ii), $\left|\dfrac{c-a}{b-a}\right| = \left|\dfrac{w-u}{v-u}\right|$

and $\arg\left(\dfrac{c-a}{b-a}\right) = \arg\left(\dfrac{w-u}{v-u}\right)$

$\Rightarrow \dfrac{AC}{AB} = \dfrac{PR}{PQ}$ and $\angle CAB = \angle RPQ$

$\Rightarrow \Delta ABC \sim \Delta PQR$

14. Sol.(d)

$|z-4| < |z-2|$

$\Rightarrow |(x-4) + iy| < |(x-2) + iy|$

$\Rightarrow (x-4)^2 + y^2 < (x-2)^2 + y^2$

$\Rightarrow -8x + 16 < -4x + 4 \Rightarrow 4x - 12 > 0$

$\Rightarrow x > 3 \Rightarrow \text{Re}(z) > 3$

15. Sol.(c)

$20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \ldots\ldots, -129\frac{1}{4}$

This is A.P. with common difference

$d_1 = -1 + \frac{1}{4} = -\frac{3}{4}$

From end $a = -129\frac{1}{4}$ and $d = \frac{3}{4}$

$T_{20} = -129\frac{1}{4} + (20-1)\left(\frac{3}{4}\right) = -129 - \frac{1}{4} + 15 - \frac{3}{4} = -115$

16. Sol.(c)

Given $S_{10} = 390 \Rightarrow \frac{10}{2}[2a + (10-1)d] = 390$

$2a + 9d = 78$

also, $\dfrac{t_{10}}{t_5} = \dfrac{15}{7} \Rightarrow \dfrac{a+9d}{a+4d} = \dfrac{15}{7} \Rightarrow 8a = 3d$ ...(ii)

Solving (i) and (ii), we have

$a = 3$ & $d = 8$

Now, $S_{15} - S_5 = \frac{15}{2}(6 + 14 \times 8) - \frac{5}{2}(6 + 4 \times 8) = 790$

17. Sol.(d)

Let r'th term of the GP be ar^{n-1}.

Given each term is arithmetic mean of next two terms.

$2a_r = a_{r+1} + a_{r+2}$

$2ar^{n} = ar^{n} + ar^{n+1} \Rightarrow \frac{2}{r} = 1 + r$

$r^2 + r - 2 = 0$

Hence we get, $r = -2$ (as $r \neq 1$)

So $S_{20} - S_{18}$ = (Sum upto 20 terms) – (Sum upto 18 terms)

$= T_{19} + T_{20}$

$T_{19} + T_{20} = ar^{18}(1+r)$ ... (i)

Putting the values $a = \frac{1}{8}$ and $r = -2$ in (i)

we get $T_{19} + T_{20} = -2^{15}$

18. Sol.(c)

$f(x+y) = f(x).f(y)\ \forall\ x, y \in N$, $f(1) = 3$ So, $f(x) = a^x$

Since, $f(1) = 3 \Rightarrow a = 3 \Rightarrow f(x) = 3^x$

$\sum_{i=1}^{n} f(k) = 3279$; $f(1) + f(2) + f(3) + \ldots\ldots\ldots f(k) = 3279$

$3 + 3^2 + 3^3 + \ldots 3^k = 3279$; $\dfrac{3(3^k - 1)}{3-1} \Rightarrow \dfrac{3^k - 1}{2} = 1093$

$3^k - 1 = 2186 \Rightarrow 3^k = 2187 \Rightarrow k = 7$

19. Sol.(c)

$x^2 \sin\theta - x(\sin\theta . \cos\theta + 1) + \cos\theta = 0.$

$\Rightarrow x^2 \sin\theta - x\sin\theta . \cos\theta - x + \cos\theta = 0.$

$\Rightarrow x\sin\theta(x - \cos\theta) - 1(x - \cos\theta) = 0.$

$\Rightarrow (x - \cos\theta)(x\sin\theta - 1) = 0.$

$\therefore\ x = \cos\theta, \operatorname{cosec}\theta, \theta \in (0, 45°)$

$\therefore\ \alpha = \cos\theta, \beta = \operatorname{cosec}\theta$

$\sum_{n=0}^{\infty} \alpha^n = 1 + \cos\theta + \cos^2\theta + \ldots \infty = \dfrac{1}{1-\cos\theta}$

$\sum_{n=0}^{\infty} \dfrac{(-1)^n}{\beta^n} = 1 - \dfrac{1}{\operatorname{cosec}\theta} + \dfrac{1}{\operatorname{cosec}^2\theta} - \dfrac{1}{\operatorname{cosec}^3\theta} + \ldots \infty$

$= 1 - \sin\theta + \sin^2\theta - \sin^3\theta + \ldots \infty.$

$= \dfrac{1}{1+\sin\theta}$

$\therefore\ \sum_{n=0}^{\infty}\left(\alpha^n + \dfrac{(-1)^n}{\beta^n}\right) = \sum_{n=0}^{\infty}\alpha^n + \sum_{n=0}^{\infty}\dfrac{(-1)^n}{\beta^n}$

$= \dfrac{1}{1-\cos\theta} + \dfrac{1}{1+\sin\theta}.$

20. Sol.(d)

Given, $2\tan^2\theta - 5\sec\theta - 1 = 0$

$\Rightarrow\ 2\sec^2\theta - 5\sec\theta - 3 = 0$

$\Rightarrow\ (2\sec\theta + 1)(\sec\theta - 3) = 0$

$\Rightarrow\ \sec\theta = -\frac{1}{2}, 3$

$\Rightarrow\ \cos\theta = -2, \frac{1}{3} \Rightarrow \cos\theta = \frac{1}{3}\ \left\{\because \theta \in \left[0, \frac{n\pi}{2}\right]\right\}$

For 7 solutions n = 13

So, $\sum_{k=1}^{13} \frac{k}{2^k} = S$ (say)

$S = \frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \ldots + \frac{13}{2^{13}}$

$\frac{1}{2}S = \frac{1}{2^2} + \frac{1}{2^3} + \ldots. + \frac{12}{2^{13}} + \frac{13}{2^{14}}$

Subtract (ii) from (i), we get

$\Rightarrow\ \frac{S}{2} = \frac{1}{2}.\frac{1-\frac{1}{2^{13}}}{1-\frac{1}{2}} - \frac{13}{2^{14}}$

$\Rightarrow\ S = 2.\left(\frac{2^{13}-1}{2^{13}}\right) - \frac{13}{2^{13}} = \frac{1}{2^{13}}(2^{14} - 15)$

Integer Type

1. Sol. (18)

Given that $|2a-1| = 3[a] + 2\{a\}$

$|2a-1| = [a] + 2a$

Case-1: $a > \frac{1}{2}$

$2a - 1 = [a] + 2a \Rightarrow [a] = -1$

$\therefore a \in [-1, 0)$ Hence Rejected

Case-2: $a < \frac{1}{2}$

$-2a + 1 = [a] + 2a \quad [\because a = I + f]$

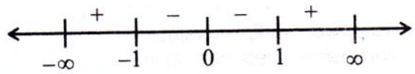
$-2(I + f) + 1 = I + 2I + 2f$

$I = 0, f = \frac{1}{4}\ \therefore a = \frac{1}{4}$

Hence $a = \frac{1}{4}$

$72\sum_{a\in S} a = 72 \times \frac{1}{4} = 18$

2. Sol. (4)



$3x^2 + x - 1 = 4|x^2 - 1|$

Case 1: If $x \in [-1, 1]$,

$3x^2 + x - 1 = -4x^2 + 4$

$\Rightarrow 7x^2 + x - 5 = 0 \because D = 141 > 0 \therefore$ Equation has two roots

Case 2: If $x \in (-\infty, -1] \cup [1, \infty)$

$3x^2 + x - 1 = 4x^2 - 4$

$\Rightarrow\ x^2 - x - 3 = 0 \because D = 13 > 0$

$\therefore$ Equation has two roots

So, total 4 roots.

3. Sol. (8)

Since, $2x^2 + (a-10)x + \frac{33}{2} = 2a$ has real roots,

$\therefore\ D \geq 0\ \Rightarrow (a-10)^2 - 4(2)\left(\frac{33}{2} - 2a\right) \geq 0$

$\Rightarrow\ (a-10)^2 - 4(33 - 4a) \geq 0 \Rightarrow a^2 - 4a - 32 \geq 0$

$\Rightarrow\ (a-8)(a+4) \geq 0 \Rightarrow a \leq -4 \cup a \geq 8$

$\Rightarrow\ a \in (-\infty, -4] \cup [8, \infty)$

4. Sol. (2)

The given equation is

$x^2 - 8kx + 16(k^2 - k + 1) = 0$

$\because$ Both the roots are real and distinct.

$\therefore\ D > 0 \Rightarrow (8k)^2 - 4 \times 16(k^2 - k + 1) > 0$

$\Rightarrow\ k > 1$

$\because$ Both the roots are greater than or equal to 4

$\therefore\ \alpha + \beta > 8$ and $f(4) \geq 0$

$\Rightarrow\ k > 1$

and $16 - 32k + 16(k^2 - k + 1) \geq 0$

$\Rightarrow\ k^2 - 3k + 2 \geq 0 \Rightarrow (k-1)(k-2) \geq 0$

$\Rightarrow\ k \in (-\infty, 1] \cup [2, \infty)$

Combining (i), (ii) and (iii), we get $k \geq 2$

$\therefore$ Smallest value of $k = 2$.

5. Sol. (9)

$\frac{S_7}{S_{11}} = \frac{6}{11} \Rightarrow \frac{\frac{7}{2}[2a + 6d]}{\frac{11}{2}[2a + 10d]} = \frac{6}{11} \Rightarrow a = 9d$

$a_7 = a + 6d = 15d$

Given $130 < a_7 < 140$

$\Rightarrow\ 130 < 15d < 140 \Rightarrow d = 9$

[Since d is a natural number because all terms are natural numbers.]